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COMMENTS ON THE PAPER: THE EFFECT OF EXTERNAL DAMPING ON THE STABILITY OF BECK'S COLUMN [1]

THE authors have considered stability of a cantilever under a tangential, compressive end-load when external damping is also present. As has been shown in [2, 3], the external damping has a stabilizing effect in this problem. The authors, moreover, have shown that the critical (flutter) load approaches a *finite* value as the value of external damping becomes large. They then state that this effect "seems to have escaped previous investigators ...".

This statement is not true.

The fact that certain damping mechanism—especially external damping—may lead to such a limiting result has been known in aeroelasticity problems, for examine in the case of the bending–torsional flutter of a swept wing in a high-density, low-speed flow [4]. As is shown by Prasad *et al.* in [4], for certain values of the angle of sweep and the aspect ratio, the flutter load parameter admits *finite* limiting values as the mass-density of the surrounding fluid becomes large, i.e. as the external damping becomes large;† see curves corresponding to $\varepsilon = 50, 25$ of Fig. 2 in Ref. [4]. An additional fact, which is shown in Fig. 3 of Ref. [4], is that the corresponding critical frequency in such cases approaches zero with the increase in damping. This is not noted by Professors Plaut and Infante because they do not calculate the limiting critical frequency. This fact, and other results obtained by these authors, can be easily arrived at by trivial specialization of results in Ref. [3]. Since, in the present case, internal damping and Coriolis forces are zero, we set $\delta = \beta = 0$, $\nu = 1$, $\gamma \equiv \xi$ and $F^2 \equiv p$ in equation (17)‡ of Ref. [2], to obtain

$$\omega^4 - iP_1\omega^3 - P_2\omega^2 + iP_3\omega + P_4 = 0, \tag{1}$$

where

$$\begin{aligned} P_1 &= 4\xi, P_2 = 4\xi^2 + (\omega_1^2 + \omega_2^2) + p(b_{11} + b_{22}) \\ P_3 &= 2\xi[\omega_1^2 + \omega_2^2 + p(b_{11} + b_{22})] \\ P_4 &= p^2[b_{11}b_{22} - b_{21}b_{12}] + p[\omega_1^2 b_{22} + \omega_2^2 b_{11}] + \omega_1^2\omega_2^2, \end{aligned} \tag{2}$$

and where the constants ω_1^2, ω_2^2, b_{11}, b_{22}, b_{12} and b_{21} are given in [3, p. 1279]. At the threshold of stability, ω is real and from (1) we obtain

$$\omega^2 = \frac{P_3}{P_1} \quad \text{and} \quad \left(\frac{P_3}{P_1}\right)^2 - P_2\frac{P_3}{P_1} + P_4 = 0 \tag{3}$$

which define the critical values of ω and p. Substitution from (2) into (3) yields the critical values of ω and p in terms of ξ. In particular, as $\xi \to \infty$ we obtain

$$\omega^2 = \tfrac{1}{2}[p(b_{11} + b_{22}) + \omega_1^2 + \omega_2^2] \tag{4}$$

† Note that the parameter μ in Fig. 2 of Ref. [4] is inversely proportional to the density of the fluid. Thus large values of damping correspond to small values of μ.

‡ This equation corresponds to a two-term Galerkin approximation. Note that a two-term Galerkin approximation in [4] yields a similar equation.

and

$$p(b_{11}+b_{22})+\omega_1^2+\omega_2^2 = 0. \tag{5}$$

Equation (5) yields

$$p_{cr} = -\frac{\omega_1^2+\omega_2^2}{b_{11}+b_{22}} \approx 40{\cdot}0$$

as compared with 39·4 obtained by the authors, using a two-term modal approximation; a more accurate result can be obtained if we use equations (9) and (12) of Ref. [3]. The important fact, however, is that if (5) is satisfied, then from (4) the critical frequency is zero, i.e. *there is no motion, and hence no flutter*. Of course for ξ large but finite, the critical frequency is small but non-zero, while p_{cr} may be very close to its limiting value.

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REFERENCES

[1] R. H. PLAUT and E. F. INFANTE, *Int. J. Solids Struct.* **6**, 491–496 (1970).
[2] H. LEIPHOLZ, Über den Einfluss der Dämpfung bei nichtkonservativen Stabilitätsproblemen elastischer Stäbe. *Ing.-Arch.* **33**, 308 (1964).
[3] S. NEMAT-NASSER, S. N. PRASAD and G. HERRMANN, Destabilizing effect of velocity-dependent forces in nonconservative continuous systems. *AIAA Jnl* **4**, 1276 (1966).
[4] S. N. PRASAD, S. NEMAT-NASSER and G. HERRMANN, Bending–torsional flutter of a swept wing in a high-density, low-speed flow. *AIAA Jnl* **5**, 316 (1967).